http://comet.lehman.cuny.edu/*fitting*/bookspapers/pdf/papers/FOLP.pdf

Forbus, K. D. (2008). Qualitative Modeling. In: Handbook of Knowledge Representation. F. van Harmelen, V. Lifschitz and B. Porter (eds.). pp. 361-393.

Frommberger, L. (2008). Learning to behave in space: a qualitative spatial Representation for robot navigation with reinforcement learning. *In:*International Journal on Artificial Intelligence Tools Vol. 17, No. 3. pp. 465–482 © World Scientific Publishing Company.

Galton, A. P. (2009). Spatial and Temporal Knowledge Representation. Earth Sci Inform. 2:169-187. Springer-Verlag.

Muller, P. (1998b). A qualitative theory of motion based on spatiotemporal primitives. Principles of Knowledge Representation and Reasoning.

Randell, D. A., Cui, Z. and Cohn, A. G. (1992). A Spatial logic based on region and connection, Proceeding of 3rd International Conference on Knowledge Representation and Reasoning, Morgan Kaufmann, Los Altos, pp. 55-66.

Renz, J. and Nebel, B. (2007). Qualitative spatial reasoning using constraint calculi. In Handbook of spatial logics, Springer.

Shakarian, P., Dickerson, J. P. and Subrahmanian, V. S. (2011). Adversarial Geospatial Abduction Problems. ACM Tansactions on Intelligent Systems and Technology, Vol. No. , 20.

Wolter, F., and Zakharyaschev, M. (2000a). Spatial reasoning in RCC-8 with Boolean region terms. In Horn, W. (Ed.), Proceedings of the 14th European Conference on Artificial Intelligence (ECAI 2000), pp. 244–248. IOS Press.

Wolter, F., and Zakharyaschev, M. (2000b). Spatio-temporal representation and reasoning based on RCC-8. In Cohn, A., Giunchiglia, F., & Seltman, B. (Eds.), Proceedings of the 7th Conference on Principles of Knowledge Representation and Reasoning (KR2000), pp. 3–14. Morgan Kaufmann.

Wolter, F., and Zakharyaschev, M. (2002). Qualitative spatio-temporal representation and reasoning: a computational perspective. In Lakemeyer, G., & Nebel, B. (Eds.), Exploring Artificial Intelligence in the New Millenium, pp. 175–216. Morgan Kaufmann.

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Introducing The Spatial Qualification Problem and Its Qualitative Model

Bassey, Patience C.
Department of Computer Science
University of Uyo
Uyo, Nigeria
imecb@yahoo.com

Akinkunmi, B.O.
Department of Computer Science
University of Ibadan
Ibadan, Nigeria
ope34646@yahoo.com

ABSTRACT

Spatial qualification problem is the impossibility of knowing an agent's presence at a particular place at a certain time to be involved in an action or be participant in an event. The problem of spatially qualifying an intelligent agent requires commonsense reasoning which is qualitatively represented in qualitative spatial reasoning, a sub-field of knowledge representation and reasoning. In this paper, we present an overview of the spatial qualification problem and the qualitative formalism for reasoning with the problem. Existing spatial and temporal calculi for reasoning were combined and reused in the definition and axiomatization of basic concepts in the formalism. Quantified Modal Logic was seen to be suitable for the qualitative reasoning about these spatial concepts. The resulting spatial qualification logic (Alibi Logic) would be applicable in domains that require investigation of the problem of spatial qualification.

Keywords: Spatial qualification problem, Commonsense reasoning, Qualitative reasoning, Possible world semantics, Quantified modal logicc

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1. INTRODUCTION

In spatial domains, the problems of describing and identifying an object, scene and route are common. Attempts to solve these problems involve direct abstraction of the needed knowledge about the world and its properties such as actual size, weight and distance as seen in most formalisms. It also involves the use of quantitative approaches and giving of precise and predicted results. This approach is too expensive as vagueness, uncertainty and granularity remains a problem in spatial and temporal domains. Spatial knowledge apart from being vague and incomplete (Galton, 2009; Cohn and Renz, 2008), is continuous, that is, it changes with respect to time. These categories of problems involve commonsense knowledge (Cohn, 1999) which is best solved by employing qualitative reasoning. A typical case of this category of problems is that of investigating spatial qualification.

Formalisms where the investigation of spatial qualification in real life scenarios has been done through deep reasoning process made use of probabilistic and fuzzy approaches. These approaches though it may lead to desired goals are quantitative. Working with large sets of data is very expensive. Also, these real world problems involve unproven ideas or assumed possibilities proposed for further investigation. Formulating most of these problems using classical logics where only the truth value of a formula is determined but not the way, mode and state of the truth of a formula, will not lead to a logical conclusion.

Spatial qualification in our context is the possibility that an agent could be present at a particular place at a certain time given prior antecedents. Reasoning about spatial qualification involves an agent's movement from one region of space to another and the rate of change in time. Thus our research questions: *Given prior antecedent to have been present at or absent from the scene of incidence under investigation, is it possible for the agent to have been at the scene of incidence at a certain time?*

African Journal of Computing & ICT

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To have an answer to the above question, there is need to further ask: *can spatial knowledge of this sort be formally represented or is there a suitable language to handle incomplete/uncertain knowledge of this sort? Can the representation be used in reasoning to reach the conclusion of "possibility" or "impossibility"? Can the validity of the formalism be proven?* Formalizing spatial qualification requires the use of a non-classical logic such as modal logic due to the efficacy of its modalities in handling the "possible worlds" concepts.

The applicability of the spatial qualification logic is very promising in domains with high demand for reasoning such as:

- *Alibi Reasoning*: In a case where an accused person gives an alibi, to investigate the given alibi to be true that there is no possibility of the accused to be present at the scene of the incidence to be involved in the crime.
- *Homeland Security*: In a case of an ATM Fraud, the model if built into the ATM machines can help to investigate the possibility of presence of an account holder at certain locations to carry out multiple transactions that are spatially questionable due the time difference between the repeated transactions.
- *Planning*: In planning, one needs to work out the feasibility of having an agent carry out an action at some future time, given its current location e.g. "I need to deliver a truck of oranges in Lagos in the next twenty minutes. I am now in Ibadan which is about 2 hours from Lagos."

The aim of this research is to formalize the logic of spatial qualification with respect to time using the techniques of qualitative modeling (Forbus, 2008; Cohn and Renz, 2008). Our formalism will provide a logical framework for investigating the problem of spatial qualification. The achievement of the above aim will result from the successful performance of objectives such as: deciding an appropriate language used for our logical theory and describing the axioms and derivation rules for our theory. The resulting formalized model (otherwise known as an alibi reasoner) is deemed fit for the investigation of any spatial qualification problem in the several domains.

The rest of the paper is organized as follows. Section 2 gives us an insight to the related literature. Section 3 discusses the methodology used in the formalization of the logic. The logic of spatial qualification is modeled in section four with parameters used and the outcome clearly represented using appropriate logical language in section 4. Section 5 gives the conclusion of the paper.

2. LITERATURE REVIEW

Attempts to represent spatial knowledge have to do with the several views about space. Representing space as a concept, made researchers to start viewing space in diverse ways. Casati and Varsi (Casati and Varzi, 1997) presented two of the commonsense view of space: Newtonian and Leibniz view. Newtonian's view of space is that space is an individual entity in its own right independent of whatever entities may inhabit it. While Leibniz's view contended with it by saying that "there is no way of identifying a region of space except by referencing what is or could be located or take place at that region." Reasoning with space requires categorization of the granularities of space and their relationship where several attempts to categorize 'place' as it relates with other spatial concepts as neighbourhood, region, district, area and location have been made (Bennett and Agarwal, 2007).

The need to express location information about objects in space calls for simplifying the mathematical concepts by approximately referring to points without measure, that is, without employing the full power of mathematical topology, geometry and analysis (Asher and Vieu, 1995). This approach is contrary to the poverty conjecture by Forbus, Nielson and Faltings: "there is no purely qualitative, general purpose kinematics" (Forbus, 2008). They concluded by suspecting that the space of representations in higher dimensions is sparse and for spatial reasoning, nothing less than numbers will do.

In an attempt to refute the poverty conjecture, increased researches in Qualitative Spatial Reasoning (QSR) has addressed different concepts of space including topology, orientation, shape, size and distance (Cohn, 1999). Qualitative reasoning allows people to draw useful conclusions about physical world without equations. It also allows one to work with far less data, than would be required when using traditional, purely quantitative methods. Frommberger (Frommberger, 2008) pointed out that the use of this representation empowers the agent to learn a certain goal-directed navigation strategy faster compared to metrical representations, and also facilitates reusing structural knowledge of the world at different locations within the same environment. Cohn pointed out that QSR is potentially useful, and that there may be many domains where QR alone is insufficient (Cohn, 1999). This called for the addition of qualitative non-topological information like orientation (Freksa, 1992), distance, size and shapes to the topological relations (Randell et al, 1992). The RCC-8 notations and their meaning are as described in the table below.

African Journal of Computing & ICT

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Table 1. The RCC-8 Notations and Meanings

S/No.	Notation	Meaning
1.	$EQ(l_1,l_2)$	l_1 Equally connected with l_2
2.	$TPP(l_1,l_2)$	l_1 is a tangential proper part of l_2
3.	$TPP(l_2,l_1)$	l_2 is a tangential proper part of l_1
4.	$NTPP(l_1,l_2)$	l_1 is not a tangential proper part of l_2
5.	$NTPP(l_2,l_1)$	l_2 is not a tangential proper part of l_1
6.	$DC(l_1,l_2)$	l_1 has a disjoint connection with l_2
7.	$EC(l_1,l_2)$	l_2 is externally connected with l_1
8.	$PO(l_1,l_2)$	l_1 is partially overlapping with l_2

The effectiveness of these qualitative relations is fully employed as models that have these combinations were also created (Muller, 1998; Erwig et al, 1999; Bennett et al; 2000). But these qualitative models are yet to address the qualification problem with respect to space. Attempts to address the spatial qualification problem made use of probabilistic and fuzzy approaches. Possible worlds here are arbitrary worlds of equally divided grids of location with directional states of randomly assigned values (Dean et al., 1993) and set of points with the reward function used to approximately assign weights to the points (Shakarian et al., 2011). In our work we view space as region rather than considering geometric points and this allows reasoning without any randomly assigned value.

3. METHODOLOGY

A combined approach that employs both spatial and temporal formalisms as its own formalism is adopted in our formal theory. The formalism makes use of certain existing spatial and temporal calculi for reasoning. Some of these calculi have been stated and defined in the literature. Interestingly to us, the Region Connection Calculus (RCC-8) and some temporal calculi which is either point based or interval based were not left out. The definitions of the RCC-8 relations (Wolter and Zakharyaschev, 2000a, 2000b, 2002; Randell et al, 1992), which is based on the region connection relation, C, for the definition from the literature of the eight disjoint pair of relations are as follows:

Def1: $\forall l\ C(l,l)$

Def2: $\forall l,l_1\ (C(l,l_1) \rightarrow C(l_1,l))$

Def3: $DC(l,l_1) \equiv \neg C(l,l_1)$

Def4: $P(l,l_1) \equiv \forall z\ (C(x,l) \rightarrow C(z,l_1))$

Def5: $EQ(l,l_1) \equiv P(l,l_1) \wedge P(l_1,l)$

Def6: $O(l,l_1) \equiv \exists z(P(z,l) \wedge P(z,l_1))$

Def7: $PO(l,l_1) \equiv O(l,l_1) \wedge \neg P(l,l_1) \wedge \neg P(l_1,l)$

Def8: $EC(l,l_1) \equiv C(l,l_1) \wedge \neg O(l,l_1)$

Def9: $PP(l,l_1) \equiv P(l,l_1) \wedge \neg P(l_1,l)$

Def10: $TPP(l,l_1) \equiv PP(l,l_1) \wedge \exists z\ (EC(z,l) \wedge EC(z,l_1))$

Def11: $NTPP(l,l_1) \equiv PP(l,l_1) \wedge \neg\exists z\ (EC(z,l) \wedge EC(z,l_1))$

The defined region connection relations are re-used in our logic to define the *Regionally_part_of* and the *Regionally_disjoint* relations as follows.

Def12: $\forall l,l_1\ Regionally\_part\_of(l,l_1) \equiv EQ(l,l_1) \vee TPP(l,l_1) \vee TPP(l_1,l) \vee NTPP(l,l_1) \vee NTPP(l_1,l)$

Def13: $\forall l,l_1\ Regionally\_disjoint(l,l_1) \equiv DC(l,l_1) \vee EC(l,l_1) \vee PO(l,l_1)$

Two representational languages are combined to obtain a suitable representational language that will help to reason qualitatively about spatial concepts necessary for investigating the spatial qualification problem. Hence, the resulting representational language combines First-Order logic because of its expressiveness with the modal operators of Modal logic.

Using a quantified (First-Order) modal logic (Fittings, 1998) leads to a new kind of semantic problem however. The literature has a good number of papers trying to define a definite semantics for quantified modal logic. One of the major problems in defining the semantics of quantified modal logics is the problem of having varying domains for different worlds within the framework of the possible world semantics. Because the individuals of interest in our domain remain the same, we are assuming that the objects in the domain remain exactly the same as one move from one possible world to another. Consequently, in our logic, the following Barcan's axiom in (i) and (ii) hold:

Vol 6. No. 1, March 2013
African Journal of Computing & ICT

www.ajocict.net

$\Box\forall x.\ P(x) \Leftrightarrow \forall x.\Box P(x)$ (i)

or

$\Diamond\exists x.P(x) \Leftrightarrow \exists x.\Diamond P(x)$ (ii)

Thus the structure of the Kripke model/Possible World Semantics (PWS) (Fittings, 2008) is best used to semantically explicate the model structure or the formalism for our theory. A possible world is a universe in contrast with reality. It is also a region indexed with time. Kripke structure is defined by a triplet, M = (W, R, V) where W is a non empty set of possible worlds, $R \subseteq W \times W$ is the accessibility relation and V: (Prop $\times$ W) $\rightarrow$ (true, false) is the valuation function. The meaning of the standard logical operators: $\wedge, \vee, \neg, \Rightarrow, \Leftrightarrow$ and the quantifiers $\forall$ and $\exists$ are as defined in the model semantics for First-Order logic. The standard modal operators "necessity" $\Box$ and "possibility" $\Diamond$ are as defined in the Kripke semantics. Something is *necessarily* true in our current world if it is true in all the worlds accessible from the current world. Something is *possibly* true in the current world if it is true in some world accessible from the current world.

Our formalization is based on the qualitative modeling approach and the resulting system of axioms will be viewed in light of the Reified First-Order Predicate logic in which state propositions are treated as individuals and the standard modalities i.e. possibility and necessity are treated as properties of states.

4. THE QUALITATIVE MODEL OF SPATIAL QUALIFICATION

A qualitative reasoning model has been created to resolve the problem of spatial qualification. A formalization of the solution to the problem of spatial qualification based on qualitative modeling has been made. Consider an agent that was present at place or location l and at a time t. Is it possible for the same agent to be present at a difference place or location l_1 at a subsequent time t_1, given what was known? This problem may be reduced in a sense to the problem of determining whether or not the agent can travel between one place or location l to another place or location l_1 between time t and time t_1. A human reasoning agent confronted with this problem would reason using the distance between location l and l_1, and the speed or the rate at which the agent could travel. Most human agents are able to estimate how long it takes to complete a journey on a certain highway (or path). As can be affirmed by most people that this kind of reasoning is commonsense reasoning because it can be answered experientially by anyone who has traversed the highway before or it can be estimated by anyone who knows the length of the highway.

The person will use some prior knowledge of the distance and the speed limit allowed on the road. This knowledge can then be used to determine the time it will take simply by dividing the distance by the speed. It is obvious that the distance and the speed limit of the road to traverse have to be known in other to determine the minimum time it will take to traverse the road. For instance, if one wants to know how long it takes to traverse from Ibadan to Onitsha and he/she knows that it takes an hour to traverse from Ibadan to Ijebu-Ode; 2 hours from Ijebu-Ode to Ore; three hours from Ore to Benin; two hours from Benin to Onitsha, then it is possible to say that it can take minimum of eight hours to traverse from Ibadan to Onitsha. Our approach to solving this problem is based on qualitative modeling. Intelligent agents can use qualitative models to reason about quantities without having to resort to the nitty-gritty of mathematics and calculus. A particular approach that is powerful in this regard is that of discretization. The major determinants for our logic include the presence log, introduced be the *Present_at* predicate and the accessibility of the locations concerned, introduced by the *Reachable* predicate. The power of modalities of modal logic, necessarily and possibly, allows the representation of the uncertain spatial knowledge as shown in the axioms below.

$\forall x.\ l.\ t.\ Present\_at(x,l,t) \Rightarrow (\exists t_1.\ t < t_1 \Rightarrow \Diamond Present\_at(x,l,t_1))$ (iii)

Axiom (iii) gives the possibility of persistence of an agent. This states that for every agent x present at location l at some time t, it implies that it is possibly true that the same agent is present at that location at a later time t_1.

The reachability axiom that determines the possibility of presence in our logic is as defined below:

$\forall x, l_1, l_2, t_1, t_2.$
$Reachable(x, l_1, l_2, (t_1, t_2)) \Leftrightarrow t_1 < t_2 \wedge$
$(Present\_at(x, l_1, t_1) \Rightarrow \Diamond Present\_at(x, l_2, t_2))$ (iv)

The following axiom gives the underlying idea of the reachability axiom.

$\forall x, l_1, l_2, t_1, t_2.$
$Reachable(x, l_1, l_2, (t_1, t_2)) \Rightarrow (\forall t_3, t_4.\ t_3 < t_4 \wedge$
$((t_4 - t_3) = (t_2 - t_1)) \Rightarrow Reachable(x, l_1, l_2, (t_3, t_4)))$ (v)

In discretization quantities are divided into chunks. And the solutions to our problems can be deduced from the solutions to the smaller versions of the problem. For example, if an agent being at location l_1 at time t_1 implies s/he can be in location l_2 at a later time t_2, and an agent being at location l_2 at time t_2 implies he can be at location l_3 at a later time t_3 and l_3 is farther from l_1 than l_2 is, then x being present at l_1 at time t_1 implies x can be present at l_3 at time t_3. This is represented in axiom (vi) below:

African Journal of Computing & ICT

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$\forall x, l_1, l_2, l_3, t, t_2, t_3.$
$Reachable(x, l_1, l_2, (t, t_2)) \wedge Reachable(x, l_2, l_3, (t_2, t_3))$
$\Rightarrow Reachable(x, l_1, l_3, (t, t_3)).$ (vi)

Also, the absence of the agent can be inferred following the axiom below:

$\forall x, l, l_1, t.$
$(Present\_at(x, l, t) \wedge Regionally\_disjoint(l, l_1))$
$\Rightarrow \neg\Diamond(Present\_at(x, l_1, t)$ (vii)

The *Regionally_disjoint* used in axiom (vii) follows from the definition in Def13.

With the above system of axioms, our logical theory should be able to make inferences that lead to the conclusion that it is possible or not possible for an object in a particular world, W_1 at a certain time to reach another world, W_2. For any of these conclusions to be met, several logical axioms based on some stated lemma and definitions about geographic space are required. The composition of definitions of the topological relations (RCC-8 relations) and the modalities with First-Order logic gives birth to our spatial qualification (alibi) logic. Quantified (First-Order) Modal logic for reasoning with this reasoning problem is presented as a system of axioms. These logical axioms for inferring the possibility of an agent's presence at a particular location at a certain time are based on qualitative reasoning. Out of the scope of this paper is the formal semantics and the syntax of the logic presented to clarify the fact that our first-order modal logic is a fixed domain logic. In other words the domain remains the same as one reasons from one possible world to another. With this system we argue that logic of presence such as ours satisfies all the properties of an S4 system of modal axioms which includes:

K: $\Box(\phi \Rightarrow \psi) \Rightarrow (\Box\phi \Rightarrow \Box\psi)$; T: $\Box\phi \Rightarrow \phi$ and 4: $\Box\psi \Rightarrow \Box\Box\psi$.

5. CONCLUSION

The possibility of an agent to be present at a particular place at a certain time is viewed as a possible world in our problem domain. This means that there is transition between some or all the possible worlds in the set. Some of these transitions may be possible while some may not. Our interest in this problem is borne out of the fact that the solution to this problem has many potential applications. Investigators in application domains like criminology, homeland security, planning, etc. will find our logical theory a useful companion required to reach possible conclusions about their investigations. This serves as an abstraction mechanism in an aspect of the formal ontology for the Semantic Web.

Our logic treats any fact we know as something that remains permanently true. As such if we know that an agent is present at a location l at time t, then that fact is always true. We state in (viii) thus:

$\forall x, l, t.\ Present\_at(x, l, t) \Rightarrow \Box Present\_at(x, l, t).$ (viii)

This axiom represents the persistence of truth that for every agent x present at location l at time t, it implies that it is necessarily true that every agent x is present at location l at a certain time t.

The above spatiotemporal logic answers the research questions earlier mentioned. This formal theory would be found wanting by companion systems in domains like criminology, homeland security against ATM fraud and planning. It's usefulness in fraud detection in ATM machines makes our logic a very useful logic that will give a relaxed mind especially as we are imbibing the cashless policy. This logic will also offer proofs of any given alibi such as the ones in forensic science to resolve legal issues.

Future work is on expressing and explicating the spatial concepts in light of the Possible World Semantics (Kripke's Model), analytically proving the logical system for validity using tableau proof method and integrating the logic into AI planning systems. For instance some agents cannot perform certain actions except they are spatially qualified to do so. This calls for the need for a planning system to be able to reason about spatial qualification. Further enhancement of the logic to reason about spatial qualification in a variable world is also required.

References

Asher, N. and Vieu, L. (1995). Towards a geometry of common Sense: A Semantics and a Complete Axiomatization of Mereotopology. In *Proceedings of IJCAI'95. pp. 846-852.*

Bennett, B. and Agarwal, P. (2007) Semantic Categories Underlying the Meaning of Place. In Spatial Information Theory, 8th International Conference, COSIT 2007, Melbourne, Australia, September 19-23, 2007, Proceedings (2007), vol. 4736 of Lecture Notes in Computer Science, Springer, pp. 78-95.

Bennett, B., Cohn, A. G., Torrini, P. and Hazarika, S. M. (2000). Region-Based Qualitative Geometry. Research Report Series for School of Computer Studies, University of Leeds.

Casati, R. and Varzi, A. C. (1997). Spatial Entities. In Oliviero Stock (ed.), Spatial and Temporal Reasoning, Dordrecht: Kluwer, 1997, pp. 73-96.

Cohn, A. G. (1999). Qualitative Spatial Representations. In: Proceeding of the International Joint Conference on Artificial Intelligence IJCAI-99 Workshop on Adaptive Spatial Representations of Dynamic Environment, pp.33-52.

Cohn, A. G. and Renz, J. (2008). Qualitative Spatial Representation and Reasoning. In Handbook of Knowledge Representation. F. van Harmelen, V. Lifschitz and B. Porter (eds.). pp. 551-596.

Dean, T., Kaelbling, L. P., Kirman, J. and Nicholson A. (1993). Planning with Deadlines in Stochastic domains. In Proceedings of AAAI93. http://lis.csail.mit.edu/pubs/pk/deanAAAI93.pdf.

Fittings, M. (1998). On Quantified Modal Logic. http://comet.lehman.cuny.edu/**fitting**/bookspapers/pdf/.../quantmodal.pdf.

Fittings, M. (2008). Possible World Semantics for First Order LP. http://comet.lehman.cuny.edu/*fitting*/bookspapers/pdf/papers/FOLP.pdf

Forbus, K. D. (2008). Qualitative Modeling. In: Handbook of Knowledge Representation. F. van Harmelen, V. Lifschitz and B. Porter (eds.). pp. 361-393.

Frommberger, L. (2008). Learning to behave in space: a qualitative spatial Representation for robot navigation with reinforcement learning. *In:*International Journal on Artificial Intelligence Tools Vol. 17, No. 3. pp. 465–482 © World Scientific Publishing Company.

Galton, A. P. (2009). Spatial and Temporal Knowledge Representation. Earth Sci Inform. 2:169-187. Springer-Verlag.

Muller, P. (1998b). A qualitative theory of motion based on spatiotemporal primitives. Principles of Knowledge Representation and Reasoning.

Randell, D. A., Cui, Z. and Cohn, A. G. (1992). A Spatial logic based on region and connection, Proceeding of 3rd International Conference on Knowledge Representation and Reasoning, Morgan Kaufmann, Los Altos, pp. 55-66.

Renz, J. and Nebel, B. (2007). Qualitative spatial reasoning using constraint calculi. In Handbook of spatial logics, Springer.

Shakarian, P., Dickerson, J. P. and Subrahmanian, V. S. (2011). Adversarial Geospatial Abduction Problems. ACM Tansactions on Intelligent Systems and Technology, Vol. No. , 20.

Wolter, F., and Zakharyaschev, M. (2000a). Spatial reasoning in RCC-8 with Boolean region terms. In Horn, W. (Ed.), Proceedings of the 14th European Conference on Artificial Intelligence (ECAI 2000), pp. 244–248. IOS Press.

Wolter, F., and Zakharyaschev, M. (2000b). Spatio-temporal representation and reasoning based on RCC-8. In Cohn, A., Giunchiglia, F., & Seltman, B. (Eds.), Proceedings of the 7th Conference on Principles of Knowledge Representation and Reasoning (KR2000), pp. 3–14. Morgan Kaufmann.

Wolter, F., and Zakharyaschev, M. (2002). Qualitative spatio-temporal representation and reasoning: a computational perspective. In Lakemeyer, G., & Nebel, B. (Eds.), Exploring Artificial Intelligence in the New Millenium, pp. 175–216. Morgan Kaufmann.