

## REASONING ABOUT PRODUCT DISTRIBUTION USING SPATIAL QUALIFICATION LOGIC

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### ABSTRACT

*The need for reasoning in planning domains about the preconditions that an agent needs to satisfy in order to be able to carry out an action has been recognized. There is need to clearly and easily represent how reasoning is done with agents in a fixed domain with much consideration for the location of an agent needed for a task in another location. The ability of an intelligent agent to make herself available at the location where and when an action is to be carried out is an important precondition required to assess an existing plan.*

*This work features the application of the spatiotemporal theoretic model for spatial qualification logic (SQL) to the problem of reasoning about the feasibility of time-bound distribution tasks requiring travel. In this regard, SQL axioms are applied to the domain of distribution. Examples are based on distances and times along the various routes to the hostels in the University of Ibadan which were obtained using Google Maps Distance Calculator. The paper demonstrates how SQL axioms can help infer whether or not an agent can be available to carry out a distribution task in a different location at a particular time. The resulting logical system facilitates making natural inference about the possibility of an agent's presence at a certain location for product distribution to be carried out at a certain time.*

**Keywords:** *Spatial qualification model, Qualitative reasoning, commonsense reasoning, Possible World Semantics, Plans with deadlines*

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## 1. Introduction

There are several preconditions needed in order to carry out an action. The problem of having to deal with infinitely many preconditions for an action is known in artificial intelligence (AI) as the qualification problem [1]. One of such preconditions is to determine whether or not an agent is spatially qualified to carry out that action [2]. Spatial qualification is roughly defined as the ability of an agent to make his or herself available at the designated location of an action in order to carry it out, given the agent's location constraints.

At the University of Rochester in the 1990s, researchers undertook the building of a conversationally proficient intelligent planning assistant as part of the TRAINS project [3]. The TRAINS project domain "is a transportation world with cities, rail links between them, engines, boxcars and the like to move things around and a variety of commodities" [4]. This TRAINS project proposed a solution to the problems of the domain through mixed initiative planning that involved using dialogues to carry out various kinds of plan inference.

To get an idea of the kind of plans that the TRAINS system needs to reason with, we present an abridged form of a dialog from the TRAINS domain:

*We better ship a boxcar of oranges to Bath by 8 am. There are some oranges at Corning and a boxcar at Danville. So we need an engine to move the boxcar. So we should move the engine at Avon, Engine e1, to Danville to pick up the boxcar there and move it from Danville to Corning, Load*

*up some oranges into the boxcar and then move it on to Bath.*

The kind of plans in this domain involves both the need to cover distances as well as do so within a certain deadline in order to be able to carry out an action at the destination. Not only should our boxcar of oranges arrive at Bath, it should arrive by 8 am. In order for the engine moving the boxcar full of oranges to arrive Bath by 8am, it must be fully loaded with oranges, ready to leave Corning by 7am if it takes an hour to drive between Corning and Bath. Similarly if it takes fifteen minutes to load a boxcar of oranges, the empty boxcar driven by engine *e1* must arrive Corning from Danville by 6:45 am.

It is obvious that one of the major keys to reasoning about the feasibility or otherwise of these plans is to be able to reason about the ability of the engine to make each of its assigned journeys within the specified time limit.

Our goal in this work differs from that of the TRAINS project which focused on developing an intelligent planning assistant that is conversationally proficient in natural language to predict future states of the world in the absence of complete knowledge [5], [6]. A similar work to the TRAINS project is the multitasking or teamwork communication frameworks for continuous tasks [7], [8]. Other similar works addressed routing problems with temporal consideration [9], [10], but none of these considered the agents' spatial qualification.

In this work, consideration is on the problem of spatial qualification of agents in a production distribution

domain requiring deadlines where the planner of the distribution has the knowledge of the agents' current location and time. Given the location of agents, the spatial qualification reasoner introduced in this paper will help the agent reason about the possibility of being present at another location at some other time in order to make a delivery.

The rest of the paper is organized as follows. An overview of the spatial qualification logic is given in section 2. Section 3 presents the application of the logic for reasoning in the domain of product distribution which is a domain requiring an agent moving across locations in order to achieve goals. Section 4 discusses the case study giving the results of the analysis. Section 5 discusses how an inference system for domain application can be built. Section 6 concludes the paper.

## 2. Overview of the Spatial Qualification Logic

The spatial qualification logic (*SQL*) [2] uses the quantified (first-order) modal logic as its representation language [11],[12],[13]. Each term can either be a constant symbol or variable symbols. There are three basic sorts of constants in the language. These are *Individuals*, *Location* and *Time points*. Locations in this logic denote the notion of regions in spatial logics. Apart from the predicates denoting the standard eight disjoint pairwise spatial relations from the region connection calculi (RCC-8) [14], the major predicates in the logic include *Present\_at* and *Reachable*.

The meanings of the classical logic operators are as given in the model

semantics for first order predicate logic [15] while that of the modal operators is attributed to them from the standard possible world semantics with the proposition  $\Box\phi$  meaning that  $\phi$  is true in all possible worlds accessible from the current world, and  $\Diamond\phi$  meaning that  $\phi$  is true in some world accessible from the current world. The *SQL* [2] is a 4-tuple  $\langle W, R, D, Iv \rangle$  which is built around the Kripke model [16], a triple  $\langle W, R, D \rangle$  where  $W$  is a set of possible worlds,  $R$  is the accessibility relation between pairs of worlds, and  $D$  is a definite domain from which individuals in the worlds are drawn and the interpretation function in equation (1),  $Iv$  so that for any item  $t$ :

$$Iv[t] = \begin{cases} v(t) & \text{if } t \text{ is a variable} \\ I(t) & \text{otherwise} \end{cases} \dots\dots(1)$$

where,  $v(t)$  is the variation function with variable time,  $t$ .

*SQL* is denoted by  $M, w \models \phi$ , with formula  $\phi$  being true in a world  $w$  of the model  $M$ . Thus the statement  $I[Present\_at, w] \subseteq X \times L \times T$  holds for *Present\_at* as well as for any other predicate, where  $X$  is the agent,  $L$ , the locations and  $T$ , time of presence.

Given that  $l$  is a location in space and  $l_1$  a different location in space, the ability to reason with prior knowledge and tell of the possibility of an agent to be present at a location at a certain time is strongly dependent on the present location of the agent, the location of incidence and the reachability of the locations. Reachability is built around regional connections of locations. The regional connections are defined in the RCC-8 base relations [17], [18], [19], [14] and re-used in the spatial qualification logic to define the

*Regionally\_part\_of* and the *Regionally\_disjoint* relations [20]. Other theorems in the spatial qualification logic [2] include the following.

The first axiom here states that if  $x$  is known to be present at location  $l$  at time  $t$  then that fact is known in all possible worlds.

$$T_{A1}: \forall x, l, t. \text{Present\_at}(x, l, t) \Rightarrow \Box \text{Present\_at}(x, l, t)$$

If an agent is present at a location at a certain time, it is possible for her to remain there for longer.

$$T_{A2}: \forall x, l, t. \text{Present\_at}(x, l, t) \Rightarrow (\forall t_1, t. t < t_1 \Rightarrow \Diamond \text{Present\_at}(x, l, t_1))$$

Location  $l_2$  is reachable from  $l_1$  within a certain time interval if and only if the agent being at  $l_1$  at the start of the interval makes it possible for it to beat  $l_2$  by the end of the time interval.

$$T_{A3}: \forall x, l_1, l_2, t_1, t_2. \text{Reachable}(x, l_1, l_2, (t_1, t_2)) \Leftrightarrow t_1 < t_2 \wedge (\text{Present\_at}(x, l_1, t_1) \Rightarrow \Diamond \text{Present\_at}(x, l_2, t_2))$$

Reachability is reflexive. An agent can reach a location it is already in.

$$T_{A4}: \forall x, l, t_1, t_2. t_1 > t_2 \Rightarrow \text{Reachable}(x, l, l, (t_1, t_2))$$

Reachability is symmetric. If  $l_2$  is reachable from  $l_1$  during a certain time interval, then  $l_1$  is reachable from  $l_2$  within the same interval.

$$T_{A5}: \forall x, l_1, l_2, t_1, t_2. \text{Reachable}(x, l_1, l_2, (t_1, t_2)) \Leftrightarrow \text{Reachable}(x, l_2, l_1, (t_1, t_2))$$

If a location is reachable from another within a certain time interval, then it is still reachable within another interval of equal or longer interval.

$$T_{A6}: \forall x, l_1, l_2, t_1, t_2. \text{Reachable}(x, l_1, l_2, (t_1, t_2)) \wedge \forall t_3, t_4. t_3 < t_4 \wedge (t_4 - t_3) \geq (t_2 - t_1) \Rightarrow \text{Reachable}(x, l_1, l_2, (t_3, t_4))$$

If an agent is in a location that is part of a region at time  $t$ , then it is also in that region at that time.

$$T_{A7}: \forall x, l, l_1, t. (\text{Present\_at}(x, l, t) \wedge \text{Regionally\_part\_of}(l, l_1)) \Rightarrow \text{Present\_at}(x, l_1, t)$$

If an agent is at a location  $r$  at time  $t$ , then for all regions tangential to the location, there exists a time duration within which the agent can possibly be at the later location.

$$T_{A8}: \forall x, l, t. \text{Present\_at}(x, l, t) \Rightarrow (\forall r. \text{Tangential}(l, r) \Rightarrow \exists \Delta t. \Diamond \text{Present\_at}(x, r, t + \Delta t))$$

It is not possible for an agent to be in two regionally disjoint locations at the same time.

$$T_{A9}: \forall x, l, l_1, t. (\text{Present\_at}(x, l, t) \wedge \text{Regionally\_disjoint}(l, l_1)) \Rightarrow \neg \text{Present\_at}(x, l_1, t)$$

Reachability is transitive. If  $l_2$  is reachable from  $l_1$  within an interval and  $l_3$  is reachable from  $l_2$  within a meeting interval, then  $l_3$  is reachable from  $l_1$  within the union of the two intervals.

$$T_{A10}: \forall x, l_1, l_2, l_3, t_1, t_2, t_3. \\ \text{Reachable}(x, l_1, l_2, (t_1, t_2)) \wedge \\ \text{Reachable}(x, l_2, l_3, (t_2, t_3)) \\ \Rightarrow \text{Reachable}(x, l_1, l_3, (t_1, t_3)).$$

### 3. Application on Product Distribution Domain

Our domain of application is the distribution of Coca-Cola product from the Coca-Cola Mini Depot (CCMD) to all the hostels within University of Ibadan (UI) campus. The Coca-Cola Mini Depot is known to have two pick-up vans meant for the distribution processes. This means that multi-agent actions take place at various locations and times in our problem domain. Our domain agents are mainly the two pick-up vans and the manager agent that keeps log of the action undertaken by van agents, do some reasoning to endorse new plan and re-plan based on urgent request and need from any of the hostels. The distribution process follows a star-like structure from the Coca Cola

Mini Depot (CCMD) to the 12 different hostels and from one hostel to the other following the given designated routes. The short hand for the names of the 12 hostels will be used henceforth. They are MH, TrH, TdH, QED, KH, BH, TBH, ZH, IH, QIH, NH and AH for Mellanby Hall, Trenchard Hall, Teddar Hall, Queen Elizabeth Hall, Kutti Hall, Bello Hall, Tafewa Balewa Hall, Zik (Azikiwe) Hall, Independence Hall, Queen Idia Hall, New PG Hall and Awolowo Hall respectively.

Route Designation:

Each route is assigned one bus daily. Each route is made up of three hostels as shown in Figure 1 based on their geographic locations.

The designated routes are:

- R1: CCMD → MH → TrH → TdH → CCMD
- R2: CCMD → QED → KH → BH → CCMD
- R3: CCMD → TBH → ZH → IH → CCMD
- R4: CCMD → QIH → NH → AH → CCMD

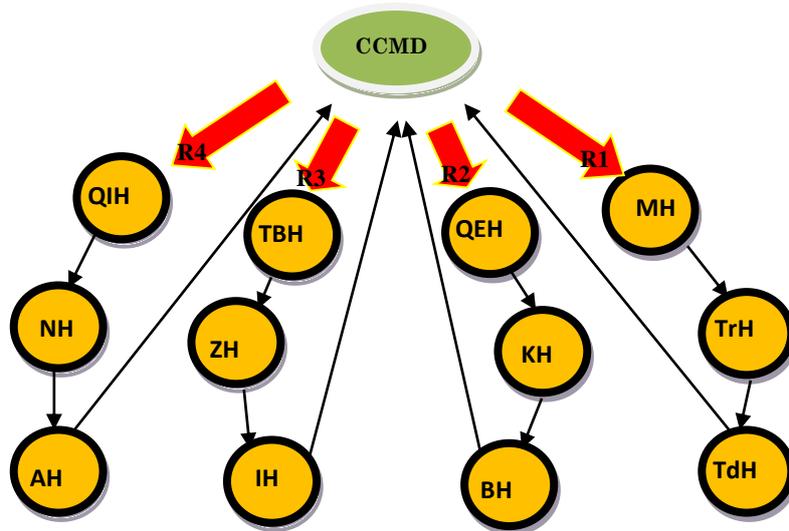


Figure 1: Figure showing designated routes, distances and time

Designated Routes	Reachable Paths	Distances (Kilometers)	Time	
			Hours	Minutes
R1	CCMD – MH	0.782	0.039	2.34
	MH – TrH	0.062	0.003	0.18
	TrH – TdH	0.143	0.007	0.42
	TdH – CCMD	0.802	0.040	2.40
R2	CCMD – QEH	1.080	0.054	3.24
	QEH – KH	0.278	0.014	0.84
	KH – BH	0.181	0.009	0.54

The distances apart from one location to another are as obtained from Google Maps of University of Ibadan [21]. To spatially qualify an agent in space requires the temporal knowledge of the world. We are assuming the speed limit of 20km/hr for the pick-up vans to traverse on roads in the university campus. Hence, the distances and the corresponding time required to cover the distances for the designated routes have been derived as shown on Table 1.

Axiom R1-3

$$\forall x, t. \text{Reachable}(x, \text{TrH}, \text{TdH}, (t, t + 0.42))$$

Axiom R1-4

$$\forall x, t. \text{Reachable}(x, \text{TdH}, \text{CCMD}, (t, t + 2.34))$$

Each of these axioms is numbered by its route number and the path it describes within the route.

For example R1-4 is the axiom describing the fourth path in route R1.

Table 1: Table showing the reachable paths in the designated routes from one location to the other, their distances and the corresponding time in University of Ibadan

Designated Routes	Reachable Paths	Distances (Kilometers)	Time	
			Hours	Minutes
R1	CCMD – MH	0.782	0.039	2.34
	MH – TrH	0.062	0.003	0.18
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	TdH – CCMD	0.802	0.040	2.40
R2	CCMD – QEH	1.080	0.054	3.24
	QEH – KH	0.278	0.014	0.84
	KH – BH	0.181	0.009	0.54
	BH – CCMD	1.053	0.053	3.18
R3	CCMD – TBH	1.143	0.057	3.42
	TBH – ZH	0.184	0.009	0.54
	ZH – IH	0.227	0.011	0.66
	IH – CCMD	1.541	0.077	4.62
R4	CCMD – QIH	1.690	0.085	5.10
	QIH – NH	0.069	0.004	0.24
	NH – AH	0.531	0.027	1.62
	AH – CCMD	2.062	0.103	6.18

For each entry on the table, there is a reachability assertion of the following kind that states that the destination is reachable from the starting location:

Axiom R1-1:

$$\forall x, t. \text{Reachable}(x, \text{CCMD}, \text{MH}, (t, t + 2.34))$$

Axiom R1-2

$$\forall x, t. \text{Reachable}(x, \text{MH}, \text{TrH}, (t, t + 0.18))$$

Note that the movement of each van to any of the designated routes follows the existing plan by the managing agent. And if any activity outside the planned activities occurs, this will call for reasoning to spatially qualify the agents. For instance, if an order is made that requires products to be delivered to BH in R2 not later than 10:00am, given that the vans have been assigned to

designated routes before the order was made, is it possible for any of the two vans to make the delivery on or before the set time?

The next section demonstrates the application of the SQL to reasoning tasks in our domain.

#### 4. Spatial Qualification Logic Case Studies

In this section we present some practical problems from the product distribution domain. There are two basic assumptions for this domain. The first assumption is that we can obtain information about the current location of the agent of interest. The second major assumption is that all the vans on our route have the same reachability times for any pairs of locations.

In each case, we reach new conclusions by forward inference and mostly modus ponens. Inferences are presented using the ground version of the axiom applied while the conclusion from each inference is made bold.

**Case 1:** Given that van1 is ready to depart for first delivery on R1 from the CCMD at 7:30am and also that van2 is ready for departure to R4 at 8:00a.m; and assuming that each van uses minimum of 30 minutes to offload the products at a particular hostel.

The logic of spatial qualification in the literature whose axioms are stated in section 2 above can be applied as it will help the managing agent to assess existing plan and determine the possibility of delivering the product to locations at specific time. With the logic, the managing agent can decide which of the vans will be available to

make the delivery without disrupting some other existing plan.

In the absence of none, the managing agent will decide which of them to use without much disruption, to ease the re-planning process.

Applying axiom  $T_{A1}$  allows us to infer that it will always be known in all worlds that van1 was at CCMD at 7.30am. We will be able to infer same for van2. Note that final inferences are marked bold in the following ground axioms.

$$\begin{aligned} &Present\_at(van1,CCMD,7:30) \\ &\Rightarrow \mathbf{\Box}Present\_at(van1,CCMD,7:30) \\ &Present\_at(van2,CCMD,8:00) \\ &\Rightarrow \mathbf{\Box}Present\_at(van2,CCMD,8:00) \end{aligned}$$

That is to say that knowing them to be present at CCMD at 7:30am also means it is possible for it to still be present at CCMD at a later time for some reasons. Also if a van is known to be at CCMD at 7.30 then by the application of  $T_{A2}$ , there is a possibility that it is still at that location at any time subsequently.

$$\begin{aligned} &Present\_at(van1,CCMD,7:30) \\ &\wedge 7:30 < 7:45 \\ &\Rightarrow \mathbf{\Diamond}Present\_at(van1,CCMD,7:45)) \\ &Present\_at(van2,CCMD,8:00) \\ &\wedge (8:00 < 7:45) \\ &\Rightarrow \mathbf{\Diamond}Present\_at(van2,CCMD,7:45)) \end{aligned}$$

Even if they are expected to depart CCMD at 7:30am, they may be there beyond that time, for instance, due to mechanical faults or fuel supply logistics.

**Case 2:** Sometimes it is necessary for trucks to move across routes. For instance if van1 is expected to make a delivery at 10:00am at MH on R1and

for some reason probably a fault reported for van2 moving to KH on R2. It is obvious that we do not have explicit information on the time it takes to move from MH to KH because we do not have information on distances across routes. However if we knew that on a certain date van2 was present at KH at 7: 30am and also at MH at 7:40am then we can infer that:

$$\forall x. \text{Reachable}(x, KH, MH, (7:30, 7:40))$$

A fully grounded axiom can be obtained by substituting van1 for x thus:

$$\text{Reachable}(\text{van1}, KH, MH, (7:30, 7:40))$$

Thus we infer that it is possible to reach MH from KH within the interval (7:30, 7:40). Thus by axiom  $T_{a5}$  we can make this inference:

$$\begin{aligned} &\text{Reachable}(\text{van1}, KH, MH, (7:30, 7:40)) \\ \Rightarrow &\text{Reachable}(\text{van1}, MH, KH, (7:30, 7:40)) \end{aligned}$$

If van1 can reach KH from MH during the interval (7:30, 7:34), then axiom  $T_{a5}$  should enable us work out when it can reach KH from MH if it leaves MH at 10:30am after delivery.

Using axiom  $T_{A6}$ , we can infer when van2 would reach KH from MH.

$$\begin{aligned} &\text{Reachable}(\text{van1}, MH, KH, (7:30, 7:40)) \wedge \\ &10:30 < 10:40 \wedge (10:30 - 10:40) \geq \\ &(7:30 - 7:40) \\ \Rightarrow &\text{Reachable}(\text{van1}, MH, KH, (10:30, 10:40)) \end{aligned}$$

**Case 3:** If there is a request for a delivery starting at TrH at 8:07am while van1 starts loading at the CCMD depot at 7:30am for thirty minutes. Will van1 be able to make the delivery?

The van1 starts loading at the CCMD depot at 7:30am and will be available for delivery at 8:00 am. The following reachability axioms are reached by a partial instantiation of the axioms from the table.

$$\forall x. \text{Reachable}(x, CCMD, MH, (8:00, 8:00+2.34))$$

$$\forall x. \text{Reachable}(x, MH, TrH, (8:00+2.34, 8:00+2.34+0.18))$$

These two axioms can become ground by substituting van1 for x.

$$\text{Reachable}(\text{van1}, CCMD, MH, (8:00, 8:00+2.34))$$

$$\text{Reachable}(\text{van1}, MH, TrH, (8:00+2.34, 8:00+2.34+0.18))$$

From these conclusions, one can apply the *SQL* axiom  $T_{A10}$  thus to reach the conclusion:

$$\begin{aligned} &\text{Reachable}(\text{van1}, CCMD, MH, (8:00, 8:00+2.34)) \\ &\wedge \text{Reachable}(\text{van1}, MH, TrH, (8:00+2.34, 8:00 \\ &+2.34+0.18)) \\ \Rightarrow &\text{Reachable}(\text{van1}, CCMD, TrH, (8:00, 8:00 \\ &+2.34+0.18)) \end{aligned}$$

Finally, by applying axiom  $T_{A6}$  and the conclusion above one can make the following inference:

$$\begin{aligned} &\text{Reachable}(\text{van1}, CCMD, TrH, (8:00, 8:00+2.34 \\ &+0.18)) \wedge 8:00 < 8:07 \wedge (8:07 - 8:00) \geq \\ &(8:00+2.34+0.18 - 8:00) \\ \Rightarrow &\text{Reachable}(\text{van1}, CCMD, TrH, (8:07, 8:00)) \end{aligned}$$

As such it is possible to infer that the van1 can be at the point of delivery before 8:07.

The next section discusses a Prolog implementation of the application described here.

## 5 Inference System

A backward chaining inferential system similar to the one built for the planner in [3] can be built for the proposed reasoning system using a logic programming paradigm. Each of the axioms in section 2 (except for  $T_{A1}$   $T_{A2}$  and  $T_{A3}$ ) can be transcribed into Prolog axioms which will enable the answering

of reachability queries by backward chaining. The only way to handle the possibility modality in  $T_{A2}$  within a Prolog framework is to first eliminate the present predicate and then introduce a new predicate possibly-present to the situation in which the modality  $\diamond$  is applied to an assertion of the type 'Present\_at'. The only problem with this approach is that it is difficult to express the semantic connection between predicates present and possibly\_present.

All the other rules can be translated into an appropriate prolog rule.

For example axiom  $T_{A4}$  expressing the fact that reachability is reflexive can be translated into a Prolog rule thus:

Prolog Rule 1

reachable(X, L, L, (T1, T2)) :-  
interval((T1, T2)).

where the following Prolog rule exists:

Prolog Rule 2

interval((T1, T2)) :-  
 $T2 > T1$ .

Similarly, Axiom  $T_{A5}$  expressing the fact that reachability is commutative can be translated into the following Prolog rule:

Prolog Rule 3

reachable(X, L1, L2, (T1, T2)) :-  
reachable(X, L2, L1, (T1, T2))  
interval((T1, T2)).

Similarly Axiom  $T_{A10}$  translates into the following Prolog rule:

Prolog Rule 4

reachable(X, L1, L3, (T1, T3)) :-  
reachable(X, L1, L2, (T1, T2))  
interval((T1, T2))  
reachable(X, L2, L3, (T2, T3))  
interval((T2, T3)).

In that case any of the conclusions in section 4 can be posed as a query and a Prolog program will be able to successfully answer. In that systems axioms formed from table 1 in section 3 such as R1-1 to R1-4 will constitute facts. For example Axiom R1-1 can be rendered in Prolog as:

reachable(X, ccmd, mh, (T, T+234))

It is important to note that time in a Prolog implementation must be rendered as an integer, for example 7:00 am is the 421<sup>st</sup> minute of the day. This makes it possible to add times and time differences.

The next section concludes the paper.

## 6. Conclusion

In the spatial qualification system considered in this paper, it is assumed that accurate information about the agent's current location are known or can be obtained through some kind of information sharing among agents. As such the following kinds of information are well established:

- Current locations of vans in the domain.
- Distances of nearby locations are known or can be obtained from interaction with other agents.

The axioms in the logic of spatial qualification can be used to infer reachability of two locations by an agent within different time frames.

Sometimes when basic information for making these inferences are unknown it is still possible to determine the reachability of any agent across locations within time frames. This can be done by studying presence logs and noting logs of the same agent and any two presence logs that are nearest in time.

Another basic assumption is that all agents have the same reachability times for any two particular locations. This assumption is valid for certain domains especially where one is reasoning about controlled worlds (e.g. industrial systems). It is not difficult to rework our logical system in order to relax this assumption. Relaxing this assumption may then make the system applicable to a world in which different agents have different travel capabilities.

The application of the *SQL* to the distribution process has shown the ability of *SQL* in assessing any existing plan for feasibility and determining whether or not re-planning is necessary in order to be able to meet the deadlines imposed by the goals of the planning agent.

The spatial qualification logic is applicable in varying domains other than planning. These include alibi reasoning, fraud detection and security control. Some of these applications are being explored.

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